

Problems for Class 1

TRUE or FALSE problems

State whether you believe the given statement is TRUE or FALSE and provide a brief argument for your answer.

1. Consider a fair six-sided die and the event of throwing a roll less than 3. The complement of this event is $\{3,6\}$.
2. Any event and its complement are mutually exclusive, but not necessarily collectively exhaustive.
3. If event A and event B are mutually exclusive and collectively exhaustive, then the complement of event A is equal to the complement of event B.
4. Consider two six-sided dice and the two events “the sum of the two dice is greater than 4” and “the sum of the two dice is less than 8”. The two events are collectively exhaustive.
5. Consider three six sided dice: one white, one green and one blue. Consider the event “the sum of the three dice equals 5”. This event is made up of 6 outcomes.
6. Consider two events A and B. We have $A \cup B = B$ and $A \cap B = A$. Then B must be the sample space.

Exercises

For the following three exercises use the sample space S defined as follows:

$$S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9, E_{10}\}$$

1. NCT 4.1

Given $A = \{E_1, E_3, E_6, E_9\}$, define $\sim A$.

2. NCT 4.2

Given $A = \{E_1, E_3, E_7, E_9\}$ and $B = \{E_2, E_3, E_8, E_9\}$

(a) What is A intersection B?

(b) What is the union of A and B?

(c) Is the union of A and B collectively exhaustive?

3. NCT 4.3

Given $\sim A = \{E_1, E_3, E_7, E_9\}$ and $\sim B = \{E_2, E_3, E_8, E_9\}$

- (a) What is A intersection B?
- (b) What is the union of A and B?
- (c) Is the union of A and B collectively exhaustive?

4. Two coins are tossed. If A is the event “two heads” and B is the event “two tails”, are A and B mutually exclusive? Are they complements?

5. NCT 4.5

A corporation takes delivery of some new machinery that must be installed and checked before it becomes available to use. The corporation is sure that it will take no more than 7 days for this installation and check to take place. Let A be the event “It will be more than 4 days before the machinery becomes available” and B the event “It will be less than 6 days before the machinery becomes available”.

- (a) Describe the event that is the complement of event A.
- (b) Describe the event that is the intersection of events A and B.
- (c) Describe the event that is the union of events A and B.
- (d) Are events A and B mutually exclusive?
- (e) Are events A and B collectively exhaustive?
- (f) Show that $(A \cap B) \cup (\sim A \cap B) = B$.
- (g) Show that $A \cup (\sim A \cap B) = A \cup B$.

6. Prove that for any event A, $0 \leq P(A) \leq 1$.