University of Bath
Department of Economics and International Development
EC10003 Introduction to Statistics
Fall 2006
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Answers to Problems for Class 1

TRUE or FALSE problems

1. FALSE.
It is {3,4,5,6}.

2. FALSE.
Any event and its complement are mutually exclusive, and collectively exhaustive.

3. FALSE
The complement of A is B and the complement of B is A.

4. TRUE
The sample space consists of the events with the sum of the two dice equal from 2 to 12. All of these events are included in either one or the other, or both of the two events given in the statement.

5. TRUE

outcome	W	G	B
1	3	1	1
2	1	3	1
3	1	1	3
4	2	2	1
5	2	1	2
6	1	2	2

6. FALSE
A must be a subset of B. B may be the sample space, but that is not necessary.

Exercises

1. NCT 4.1
$\sim A$ is the complement of event A and contains all of the sample points that are not in event A. Therefore, $\sim A = (E_2, E_4, E_5, E_7, E_8, E_{10})$.

2. NCT 4.2
(a) A intersection B contains the sample points that are in both A and B. The intersection = (E_3, E_9).

(b) A union B contains the sample points in A or B or both. The union = (E_1, E_2, E_3, E_7, E_8, E_9)

(c) A union B is not collectively exhaustive – it does not contain all of the possible sample points.

3. NCT 4.3

(a) $A \cap B = (E_4, E_5, E_6, E_{10})$

(b) $A \cup B = (E_1, E_2, E_4, E_5, E_6, E_7, E_8, E_{10})$

(c) The union of A and B is not collectively exhaustive – it does not contain all of the sample points in the sample space.

4. Let (X_1,X_2) denote the ordered pair indicating the outcome of the two coins tossed, so that X_1 indicates the outcome of the toss of the first coin and X_2 indicates the outcome of the toss of the second coin.
Then the sample space consists of the following outcomes:

$$\Omega = \{(H,H),(H,T),(T,H),(T,T)\}$$

(Note that the elements of this set are the ordered pairs).

We have:
A = {(H,H)}
B = {(T,T)}

Since $A \cap B = \emptyset$, A and B are mutually exclusive.

Since $\sim A = \{(H,T),(T,H),(T,T)\} \neq B$, A and B are not complements.

5. NCT 4.5

(a) The complement of event A is that it will take 4 days or less before the machinery becomes operational.
(b) The intersection of A and B will be the event that it takes 5 days before the machinery becomes operational.
(c) The union of A and B is the event of 1 day, 2 days, 3 days, 4 days, 5 days, 6 days or 7 days.
(d) A and B are not mutually exclusive because $P(A \cap B) \neq \emptyset$
(e) Yes, A and B are collectively exhaustive because they include all of the possible sample points.
(f) ($A \cap B$) is the event that it takes 5 days. ($\sim A \cap B$) is the event that it takes 4 days, 3 days, 2 days, 1 day. The union between these two events is that it takes less than 6 days (1 through 5) before the machinery is operational. This is the definition of event B, therefore $(A \cap B) \cup (\sim A \cap B) = B$

(g) $(\sim A \cap B)$ is the event that it takes 4 days, 3 days, 2 days, 1 day. Since A is the event 5 days, 6 days, 7 days, Then $A \cup (\sim A \cap B)$ will be the event of 1 through 7 days. This is the event of $A \cup B$. Therefore, $A \cup (\sim A \cap B) = A \cup B$.

6. The $0 \le P(A)$ part is the first condition of the definition of probability, so we must only prove that $P(A) \le 1$.

Suppose that $P(A) > 1$.
In any case we must have: $P(A \cup \sim A) = P(A) + P(\sim A)$
from the third condition of the definition of probability, because $A \cap (\sim A) = \emptyset$.

But we also have $P(A \cup \sim A) = P(\Omega) = 1$ from the second condition of the definition of probability.

Therefore
$P(A) + P(\sim A) = 1 \Rightarrow P(\sim A) = 1 - P(A) < 0$
because we have supposed that $P(A) > 1$. But we cannot have $P(\sim A) < 0$ because it violates the first condition of the definition. Therefore we cannot have $P(A) > 1$.