University of Bath
Department of Economics and International Development
EC10003 Introduction to Statistics
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Answers to Problems for Class 2

TRUE or FALSE problems

1. TRUE
To calculate this probability we treat the throw of a fair die like a random of experiment with 6 equally likely outcomes, of which we count the 2 that have the attribute of interest.
Then by classical probability:
P(throw less than 3) = (number of favourable outcomes) / (total number of outcomes) = 2 / 6 = 1 / 3

2. FALSE
Let X_1 denote the roll of one die and X_2 denote the roll of the other. Then:

$$P(X_1 + X_2 = 8) = P[(6,2) or (2,6) or (5,3) or (3,5) or (4,4)] = \frac{5}{36}$$

$$P(X_1 + X_2 = 7) = P[(6,1) or (1,6) or (5,2) or (2,5) or (4,3) or (3,4)] = \frac{6}{36} = \frac{1}{6}$$

3. FALSE
P(drawing marble of colour i) = 2/8 = 1/4 for i: blue, red, yellow, green.
Then, because sampling with replacement:
P(2 consecutive red marbles) = (1/4) (1/4) = 1/16 = P(first blue then green marble)

4. TRUE

$$P(2\ aces) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{13} \cdot \frac{1}{17} = \frac{1}{221}$$

By probability multiplication rule.

5. TRUE
P(King|red) = P(King and red) / P(red) = (2/52) / (1/2) = 2/26 = 1/13.

6. TRUE
There are 1,000 numbers between 0 and 999. There are 9x9x9=729 of them without a 7. Therefore there are 1,000-729=271 numbers with at least one7.

7. TRUE

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0 \neq P(A)$$

Since A is not the empty set.

Exercises

NCT 4.9

The total number of outcomes in the sample space, $N = C_3^{10} = \frac{10!}{3!(10-3)!} = 120$.

The number of ways to select 1 A from the 6 available, $C_1^6 = \frac{6!}{1!(6-1)!} = 6$.

The number of ways to select 2 B's from the 4 available, $C_2^4 = \frac{4!}{2!(4-2)!} = 6$.

The number of outcomes that satisfy the condition of 1A and 2B's is 6 X 6 = 36. Therefore, the probability that a randomly selected set of 3 will include 1A and 2 B's is $P_A = \frac{N_A}{N} = \frac{C_1^6 C_2^4}{C_3^{10}} = \frac{6X6}{120} = .30$

NCT 4.13

a. $P(A) = P(5 \text{ days} \cup 6 \text{ days} \cup 7 \text{ days}) = .41 + .20 + .07 = .68$

b. $P(B) = P(3 \text{ days} \cup 4 \text{ days} \cup 5 \text{ days}) = .08 + .24 + .41 = .73$

c. $P(\overline{A}) = P(3 \text{ days}, 4 \text{ days}) = .08 + .24 = .32$

d. $P(A \cap B) = P(5 \text{ days}) = .41$

e. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .68 + .73 - .41 = 1.0$

NCT 4.17

a. $P(A) = .39 + .23 + .15 + .06 + .03 = .86$

b. $P(B) = .14 + .39 + .23 + .15 = .91$

c. $P(\overline{A}) = 1 - P(A) = 1 - .86 = .14$

d. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .86 + .91 - .77 = 1.00$

e. $P(A \cap B) = .39 + .23 + .15 = .77$

f. Check if $P(A \cap B) = 0$. Because $P(A \cap B) = .77 \neq 0$, A and B are not mutually exclusive

g. Yes, because $P(A \cup B) = 1$, A and B are collectively exhaustive

NCT 4.31

If A is the event 'no graduate student is selected', then the number of combinations of 3 chosen from 6 is $C_3^6 = 6!/3!3! = 20$ and the number of combinations of 0 objects chosen from 2 is $C_0^2 = 2!/0!2! = 1$. $P(A) = 1/20 = .05$

NCT4.35

a. ${}_7P_2 = \frac{7!}{5!} = 42$

b. ${}_6P_1 = \frac{6!}{5!} = 6$

c. ${}_6P_1 = \frac{6!}{5!} = 6$

d. Probability of being chosen as the heroine = 6 chances out of 42 = 1/7. Since there are seven candidates for 1 part – a randomly chosen candidate would have a 1 in 7 chance of getting any specific part.
e. Since being chosen as the heroine or as the best friend are mutually exclusive, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 1/7 + 1/7 – 0 = 2/7. Since there are seven candidates for 2 parts – a randomly chosen candidate would have a 2 in 7 chance of getting apart.

NCT 4.36
a. $C_2^5 = 5!/2!3!$ = 10, $C_4^6 = 6!/4!2!$= 15. Since the selections are independent, then there are (10)(15) = 150 possible combinations.
b. P(select a brother who is a craftsman) = $C_1^4/10 = [4!/1!3!]/10 = 4/10$. Because there are only 5 craftsmen, once a brother has been selected as a craftsman there are only four ways to fill the second craftsman spot on the work crew. P(select a brother who is a labourer) = $C_3^5/15 = [5!/2!3!]/10$ = 10/15. Multiply the two probabilities together to find their intersection: (4/10)(10/15) = .2667
c. The probability of the complements is 1 minus the probability of the event. Therefore, P(not selecting a brother who is a craftsman = 1 – 4/10 = 6/10. P(not selecting a brother who is a labourer) = 1-10/15 = 5/15. Multiply the two probabilities together to find their intersection: (6/10)(5/15) = .20

NCT 4.42
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C)$ = .02 + .01 + .04 - .0002 - .0008 - .000 = .069

NCT 4.44
Let
A : watch a TV program oriented to business and financial issues
B : read a publication.
Then $P(A)$ = .18, $P(B)$ = .12 and $P(A \cap B)$ = .10
a. $P(B|A) = P(A \cap B) / P(A)$ = .10/.18 = .5556
b. $P(A|B) = P(A \cap B) / P(B)$ = .10/.12 = .8333

NCT 4.52
Let
A : new customer
B : call to a rival service customer.
Then
$P(A)$ = .15, $P(B)$ = .6 and $P(B|A)$ = .8.
$P(A|B) = P(A \cap B/ P(B)$
where $P(A \cap B) = P(B|A)P(A)$.
Then [(.8)(.15)]/.6 = .2

NCT 4.93

a. False: Given that event B occurs has changed the sample space, hence, the revised probability may be less
b. False: If the probability of its complement is zero, then the event and its complement will be dependent events
c. True: This follows because the probabilities of events must be non-zero. By the Multiplicative Law of Probability: $P(A|B)P(B) = P(A \cap B)$. Since $0 \le P(B) \le 1$, then the $P(A|B) \ge P(A \cap B)$.
d. False: this statement is true only when the two events are independent.

11. Note that:
P(Emma and Josh have different blood types) =

=1- P(Emma and Josh have the same blood type)

Let E_X denote the event that Emma has blood type X, and J_X denote the event that Josh has blood type X.

Then:

P(Emma and Josh have the same blood type) =
$= P[(E_A \cap J_A) \cup (E_B \cap J_B) \cup (E_{AB} \cap J_{AB}) \cap (E_O \cap J_O)] =$

$= P(E_A \cap J_A) + P(E_B \cap J_B) + P(E_{AB} \cap J_{AB}) + P(E_O \cap J_O) =$
(since all four intersections are mutually exclusive)
$= P(E_A)P(J_A) + P(E_B)P(J_B) + P(E_{AB})P(J_{AB}) + P(E_O)P(J_O) =$
(since the events E_X and J_X are independent for all X)
$== 0.40 \cdot 0.40 + 0.10 \cdot 0.10 + 0.05 \cdot 0.05 + 0.45 \cdot 0.45 = 0.375$

So:
P(Emma and Josh have different blood types) = 1- 0.375 = 0.625

12. Note that the question in effect asks:

What is the smallest integer n such that:

P(at least one of n children survives to age 21) $\ge$ 0.75

Assuming that the fates of the n children are independent events:

P(at least one of n children survives to age 21) =
=1 – P(all n children die before they reach 21) = 1 – $(0.80)^n$

Evaluating this probability for different n:

n	$1-(0.80)^n$
…	…
5	0.67
6	0.74
7	0.79

Therefore the smallest number of children for which the probability is at least 0.75 that at least one of them survives to adulthood is 7.