University of Bath
Department of Economics and International Development
EC10003 Introduction to Statistics
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# Answers to Problems for Class 3

## TRUE or FALSE problems

1. FALSE
P(H,W) = P(W)P(H|W) = 0.40 x 0.55 = 0.22
P(W|H) = P(W,H) / P(H) = 0.22 / 0.34 = 0.65

2. FALSE
It is an example of a random experiment.

3. FALSE
f(X) cannot be negative.

4. FALSE
E(X) = 2

5. FALSE
$P(X \leq 2.5) = 2/3$

## Exercises

1. We have:

$$\sum_{i=1}^{n}(a+bx_i) = \sum_{i=1}^{n} a + \sum_{i=1}^{n} bx_i = na + b\sum_{i=1}^{n} x_i$$

2. We have:

$$\sum_{i=1}^{n}(a+bx_i+cy_i)^2 = \sum_{i=1}^{n}(a^2 + b^2x_i^2 + c^2y_i^2 + 2abx_i + 2acy_i + 2bcx_iy_i) =$$

$$= \sum_{i=1}^{n} a^2 + \sum_{i=1}^{u}(b^2x_i^2) + \sum_{i=1}^{n}(c^2y_i^2) + \sum_{i=1}^{n}(2abx_i) + \sum_{i=1}^{n}(2acy_i) + \sum_{I=1}^{N}(2bcx_iy_i) =$$

$$= na^2 + b^2\sum_{i=1}^{n} x_i^2 + c^2\sum_{i=1}^{n} y_i^2 + 2ab\sum_{i=1}^{n} x_i + 2ac\sum_{i=1}^{n} y_i + 2bc\sum_{i=1}^{n} x_iy_i$$

NCT 4.85

Let
$A_1$ = Professor receives advertising material
$A_2$ = Professor does not receive advertising material
$B_1$ = Adopts the book
$B_2$ = Does not adopt the book
Then:
$P(A_1) = .8$, $P(A_2) = 1 - .8 = .2$, $P(B_1| A_1) = .3$, $P(B_1 \mid A_2) = .1$

and

$$P(A_1 \mid B_1) = \frac{P(B_1 \mid A_1)P(A_1)}{P(B_1 \mid A_1)P(A_1) + P(B_1 \mid A_2)P(A_2)} = \frac{(.3)(.8)}{(.3)(.8) + (.1)(.2)} = .923$$

NCT 4.106
$P(\mathrm{D|P}) = P(\mathrm{P \cap D})/P(\mathrm{P}) = P(\mathrm{P|D})P(\mathrm{D})/[P(\mathrm{P|D})P(\mathrm{D}) + P(\mathrm{P}|\bar{D})P(\bar{D})] =$
$(.8)(.08)/[(.8)(.08) + (.2)(.92)] = .2581$

NCT 4.113
a. $P(\mathrm{P})=P(\mathrm{P|HS})P(\mathrm{HS}) + P(\mathrm{P|C})P(\mathrm{C}) + P(\mathrm{P|O})P(\mathrm{O})=$
$=(.2)(.3)+(.6)(.5) + (.8)(.2)=.52$

b. $P(\overline{HS}\,|\mathrm{P}) = P((\mathrm{C \cup O}) \cap \mathrm{P})/P(\mathrm{P}) = [P(\mathrm{C \cap P}) + P(\mathrm{P \cap O})]/P(\mathrm{P}) = (.3 + .16)/.52$
$= .46/.52 = .8846$

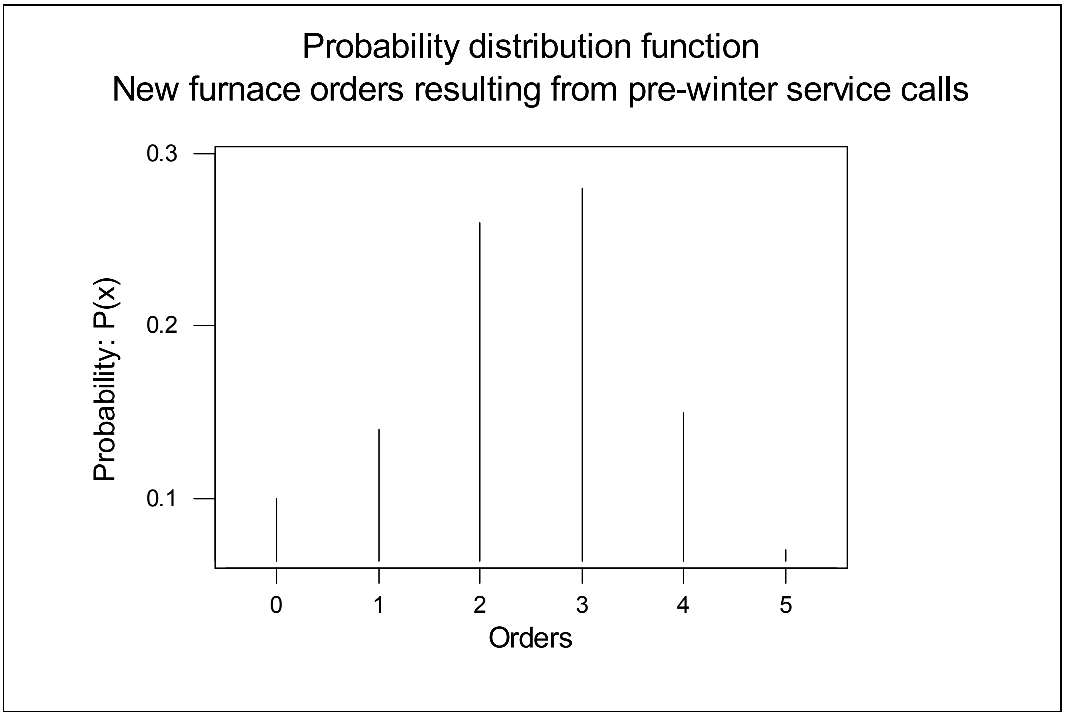
NCT 5.14
a. Cumulative probability function:

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| P(x) | .10 | .08 | .07 | .15 | .12 | .08 | .10 | .12 | .08 | .10 |
| F(x) | .10 | .18 | .25 | .40 | .52 | .60 | .70 | .82 | .90 | 1.00 |

b. $P(x \geq 5) = .08 + .10 + .12 + .08 + .10 = .48$

c. $P(3 \leq x \leq 7) = .15 + .12 + .08 + .10 + .12 = .57$

NCT 5.19
a. Probability mass function:




b. Let the cumulative distribution function of X be F(X). Then:

$$F(X)=\begin{cases} 0 & \text{for } X<0 \\ 0.10 & \text{for } 0\le X<1 \\ 0.24 & \text{for } 1\le X<2 \\ 0.50 & \text{for } 2\le X<3 \\ 0.78 & \text{for } 3\le X<4 \\ 0.93 & \text{for } 4\le X<5 \\ 1 & \text{for } 5\le X \end{cases}$$

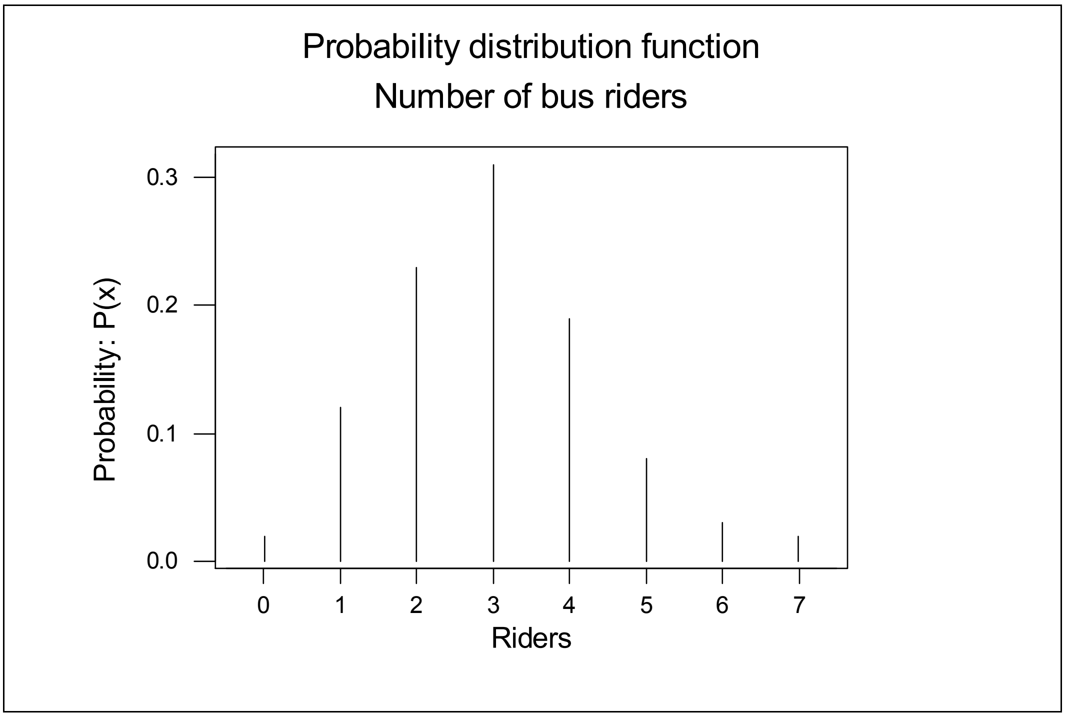
The graph of this function is a step function with the steps occurring where F(X) changes value.

c. $P(x \geq 3) = .50$

d. $\mu_x$ = 2.45 orders

e. $\sigma_x$ = 1.3592 orders

NCT 5.21
a. Probability mass function




b. Let the cumulative distribution function of X be F(X). Then:

$$F(X)=\begin{cases} 0 & \text{for } X<0 \\ 0.02 & \text{for } 0 \le X < 1 \\ 0.14 & \text{for } 1 \le X < 2 \\ 0.37 & \text{for } 2 \le X < 3 \\ 0.68 & \text{for } 3 \le X < 4 \\ 0.87 & \text{for } 4 \le X < 5 \\ 0.95 & \text{for } 5 \le X < 6 \\ 0.98 & \text{for } 6 \le X < 7 \\ 1.00 & \text{for } 7 \le X \end{cases}$$

The graph of this function is a step function with the steps occurring where F(X) changes value.

c. $P(x \ge 4) = .32$

d. $P(X < 3 \text{ (both days)}) = (.37)^2 = .1369$

e. $\mu$ = 2.99 riders $\sigma^2$ = 1.9899 $\sigma$ = 1.4106 riders

| BusRiders | P(x) | F(x) | Mean | Variance |
|---|---|---|---|---|
| 0 | 0.02 | 0.02 | 0 | 0.17880200 |
| 1 | 0.12 | 0.14 | 0.12 | 0.47521200 |
| 2 | 0.23 | 0.37 | 0.46 | 0.22542300 |
| 3 | 0.31 | 0.68 | 0.93 | 0.00003100 |
| 4 | 0.19 | 0.87 | 0.76 | 0.19381900 |
| 5 | 0.08 | 0.95 | 0.4 | 0.32320800 |
| 6 | 0.03 | 0.98 | 0.18 | 0.27180300 |
| 7 | 0.02 | 1.00 | 0.14 | 0.32160200 |
| | 1.00 | | 2.99 | 1.98990000 |
| | | | S.D. | 1.41063815 |

f. Revenue:
r = .50X, E(r) = .50(2.99) = 1.495

$\sigma_r$ = |.50|(1.4106) = .7053

NCT 5.23
(a) Let the pmf of X be *f(X)*. Then:
$f(1) = .40$
$f(2) = (.40)(.60) = .24$
$f(3) = (.40)(.60)^2 = .144$
$f(4) = (.40)(.60)^3 = .0864$
$f(x) = (.40)(.60)^{x-1}$ for x = 5, 6, …

(b) Let the cdf of X be *F*(X). Then:
F(1) = .40
F(2) = .64
F(3) = .784
F(4) = .8704
$F(x) = 1 – (.6)^x$ for x = 5, 6, …

(c) $P(X \geq 3) = 1 – P(X < 3) = 1 - .64 = .36$

NCT 5.25
$\mu$ = E(X) = 0 + .15 + 2(.19) + 3(.26) + 4(.19) + 5(.11) = 2.62 phone calls

$\sigma$ = 1.4470

11. See problem 13(a) for method.

12. See problem 13(a) for method.

13. (a) Let a simultaneous throw be denoted by $(X_1, X_2)$. Then we have:

$$P(X=1)=P[(1,1)]=\frac{1}{36}$$

$$P(X=2)=P[(1,2)or(2,1)]=\frac{2}{36}$$

$$P(X=3)=P[(1,3)or(3,1)]=\frac{2}{36}$$

$$P(X=4)=P[(1,4)or(4,1)or(2,2)]=\frac{3}{36}$$

$$P(X=5)=P[(1,5)or(5,1)]=\frac{2}{36}$$

$$P(X=6)=P[(1,6)or(6,1)or(2,3)or(3,2)]=\frac{1}{36}$$

$$P(X=8)=P[(2,4)or(4,2)]=\frac{2}{36}$$

$$P(X=9)=P[(3,3)]=\frac{1}{36}$$

$$P(X=10)=P[(2,5)o(5,2)]=\frac{2}{36}$$

$$P(X=12)=P[(2,6)or(6,2)or(3,4)o(4,3)]=\frac{4}{36}$$

$$P(X=15)=P[(3,5)o(5,3)]=\frac{2}{36}$$

$$P(X=16)=P[(4,4)]=\frac{1}{36}$$

$$P(X=18)=P[(3,6)or(6,3)]=\frac{2}{36}$$

$$P(X=20)=P[(4,5)or(5,4)]=\frac{2}{36}$$

$$P(X=24)=P[(4,6)or(6,4)]=\frac{2}{36}$$

$$P(X=25)=P[(5,5)]=\frac{1}{36}$$

$$P(X=30)=P[(5,6)or(6,5)]=\frac{2}{36}$$

$$P(X=36)=P[(6,6)]=\frac{1}{36}$$

Let the pmf of *X* be *f(X)*. Then:

$$f(X)=\begin{cases} 1/36 \text{ for } X=1,9,16,25,36 \\ 2/36 \text{ for } X=2,3,5,8,10,15,18,20,24,30 \\ 3/36 \text{ for } X=4 \\ 4/36 \text{ for } X=6,12 \\ 0 \text{ otherwise} \end{cases}$$

Let the cdf of *X* be *F(X)*. Then:

$$F(X)=\begin{cases} 0 & \text{for } X<1 \\ 1/36 & \text{for } 1\le X<2 \\ 3/36 & \text{for } 2\le X<3 \\ 5/36 & \text{for } 3\le X<4 \\ 8/36 & \text{for } 4\le X<5 \\ 10/36 & \text{for } 5\le X<6 \\ 14/36 & \text{for } 6\le X<8 \\ 16/36 & \text{for } 8\le X<9 \\ 17/36 & \text{for } 9\le X<10 \\ 19/39 & \text{for } 10\le X<12 \\ 23/36 & \text{for } 12\le X<15 \\ 25/36 & \text{for } 15\le X<16 \\ 26/36 & \text{for } 16\le X<18 \\ 28/36 & \text{for } 18\le X<20 \\ 30/36 & \text{for } 20\le X<24 \\ 32/36 & \text{for } 24\le X<25 \\ 33/36 & \text{for } 25\le X<30 \\ 35/36 & \text{for } 30\le X<36 \\ 1 & \text{for } 36\le X \end{cases}$$

$$E(X)=\sum_i x_i f(x_i)=1\cdot\frac{1}{36}+2\cdot\frac{2}{36}+3\cdot\frac{2}{36}+4\cdot\frac{3}{36}+5\cdot\frac{2}{36}+6\cdot\frac{4}{36}+8\cdot\frac{2}{36}+9\cdot\frac{1}{36}+10\cdot\frac{2}{36}+$$
$$12\cdot\frac{4}{36}+15\cdot\frac{2}{36}+16\cdot\frac{1}{36}+18\cdot\frac{2}{36}+20\cdot\frac{2}{36}+24\cdot\frac{2}{36}+25\cdot\frac{1}{36}+30\cdot\frac{2}{36}+36\cdot\frac{1}{36}=\frac{441}{36}=12.25$$

$$Var(X)=\sum_i (x_i-\mu_X)^2\cdot f(x_i)=(1-12.25)^2\cdot\frac{1}{36}+(2-12.25)^2\cdot\frac{2}{36}+(3-12.25)^2\cdot\frac{2}{36}+(4-12.25)^2\cdot\frac{3}{36}$$

$$+(5-12.25)^2\cdot\frac{2}{36}+(6-12.25)^2\cdot\frac{4}{36}+(8-12.25)^2\cdot\frac{2}{36}+(9-12.25)\cdot\frac{1}{36}+(10-12.25)^2\cdot\frac{2}{36}+$$

$$(12-12.25)^2\cdot\frac{4}{36}+(15-12.25)^2\cdot\frac{2}{36}+(16-12.25)^2\cdot\frac{1}{36}+(18-12.25)^2\cdot\frac{2}{36}+(20-12.25)^2\cdot\frac{2}{36}$$

$$+(24-12.25)^2\cdot\frac{2}{36}+(25-12.25)^2\cdot\frac{1}{36}+(30-12.25)^2\cdot\frac{2}{36}+(36-12.25)^2\cdot\frac{1}{36}=$$

$$=(126.563+210.125+171.125+204.19+105.125+156.25+36.125+10.56$$

$$10.13+0.25+15.125+14.063+66.125+120.125+276.125+162.563+630.125+564.063)/36=$$

$$=2,878.757/36=79.97$$

(b) To find the variance of Y, we must first find its expected value.

Since $Y=\sqrt{X}$ we have

$$E(Y)=E(\sqrt{X})=\sum_i \sqrt{x_i}\cdot f(x_i)=$$

$$=1\cdot\frac{1}{36}+\sqrt{2}\cdot\frac{2}{36}+\sqrt{3}\cdot\frac{2}{36}+2\cdot\frac{3}{36}+\sqrt{5}\cdot\frac{2}{36}+\sqrt{6}\cdot\frac{4}{36}+\sqrt{8}\cdot\frac{2}{36}+3\cdot\frac{1}{36}+\sqrt{10}\cdot\frac{2}{36}+$$

$$\sqrt{12}\cdot\frac{4}{36}+\sqrt{15}\cdot\frac{2}{36}+4\cdot\frac{1}{36}+\sqrt{18}\cdot\frac{2}{36}+\sqrt{20}\cdot\frac{2}{36}+\sqrt{24}\cdot\frac{2}{36}+5\cdot\frac{1}{36}+\sqrt{30}\cdot\frac{2}{36}+6\cdot\frac{1}{36}=$$

$$=\frac{25+12\cdot\sqrt{2}+10\cdot\sqrt{3}+6\cdot\sqrt{5}+8\cdot\sqrt{6}+2\cdot\sqrt{10}+2\cdot\sqrt{15}+2\cdot\sqrt{30}}{36}\cong\frac{117.328}{36}\cong 3.259=\mu_T$$

Then the variance of *Y* is:

$$Var(Y)=E[(Y-\mu_Y)^2]=E[(\sqrt{X}-\mu_Y)^2]=\sum_i (\sqrt{x_i}-\mu_Y)\cdot f(x_i)=$$

$$=(1-3.259)^2\cdot\frac{1}{36}+(\sqrt{2}-3.259)^2\cdot\frac{2}{36}+(\sqrt{3}-3.259)^2\cdot\frac{2}{36}+(2-3.259)^2\cdot\frac{3}{36}+(\sqrt{5}-3.259)^2\cdot\frac{2}{36}+$$

$$(\sqrt{6}-3.259)^2\cdot\frac{4}{36}+(\sqrt{8}-3.259)^2\cdot\frac{2}{36}+(3-3.259)^2\cdot\frac{1}{36}+(\sqrt{10}-3.259)^2\cdot\frac{2}{36}+$$

$$(\sqrt{12}-3.259)^2\cdot\frac{4}{36}+(\sqrt{15}-3.259)^2\cdot\frac{2}{36}+(4-3.259)^2\cdot\frac{1}{36}+(\sqrt{18}-3.259)^2\cdot\frac{2}{36}+$$

$$(\sqrt{20}-3.259)^2\cdot\frac{2}{36}+(\sqrt{24}-3.259)^2\cdot\frac{2}{36}+(5-3.259)^2\cdot\frac{1}{36}+(\sqrt{30}-3.259)^2\cdot\frac{2}{36}+(6-3.259)^2\cdot\frac{1}{36}=$$

$$=\frac{58.631}{36}\cong 1.63$$

(c) $P(Y\le 4.25)=P(\sqrt{X}\le 4.25)=P(X\le 18.0625)=\frac{28}{36}=\frac{7}{9}$ from the cdf of X.