University of Bath
Department of Economics and International Development
EC10003 Introduction to Statistics
Fall 2006
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Answers to Problems for Class 5

TRUE or FALSE problems

1. TRUE
P(Z<0.1) = 0.5398, and
P(Z<0.99) = 0.8389

2. FALSE
Let X be the natural logarithm of the height. Then X~N(5.14, $(2.48)^2$)
The statement in the question is equivalent to:
$$P(X \le \ln(180)) = 0.75$$
But:
$$P(X \le \ln(180)) = P(X \le 5.1930) = P(\frac{X-5.14}{2.48} \le \frac{5.1930\text{-}5.14}{2.48}) =$$
$$= P(Z \le \frac{0.053}{2.48}) = P(Z \le 0.0214) \approx 0.5080$$

3. FALSE
$$P(Y \le 3) = 1 - e^{-\frac{1}{2}3} = 1 - e^{-1.5} = 0.777$$

Exercises

NCT 6.18
a. Find Z_0 such that P(Z < Z_0) = .7, closest value of Z_0 = .52
b. Find Z_0 such that P(Z < Z_0) = .25, closest value of Z_0 = -.67
c. Find Z_0 such that P(Z > Z_0) = .2, closest value of Z_0 = .84
d. Find Z_0 such that P(Z > Z_0) = .6, closest value of Z_0 = -.25

NCT 6.22
a. $P(Z < \frac{400-380}{50}) = P(Z < .4) = .6554$
b. $P(Z > \frac{360-380}{50}) = P(Z > -.4) = F_Z(.4) = .6554$
c. The graph should show the property of symmetry – the area in the tails equidistant from the mean will be equal.
d. $P(\frac{300-380}{50} < Z < \frac{400-380}{50}) = P(-1.6 < Z < .4) = F_Z(.4) - [1-F_Z(1.6)] = .6554 - .0548 = .6006$

e. The area under the normal curve is equal to .8 for an infinite number of ranges – merely start at a point that is marginally higher. The shortest range will be the one that is centered on the z of zero. The z that corresponds to an area of .8 centered on the mean is a Z of ±1.28. This yields an interval of the mean plus and minus $64: [$316, $444]

NCT 6.24

a. $P(Z > \frac{38-35}{4}) = P(Z > .75) = 1 - F_Z(.75) = .2266$

b. $P(Z < \frac{32-35}{4}) = P(Z < -.75) = 1 - F_Z(.75) = .2266$

c. $P(\frac{32-35}{4} < Z < \frac{38-35}{4}) = P(-.75 < Z < .75) = 2F_Z(.75) – 1 = 2(.7734) – 1 = .5468$

d. (i) The graph should show the property of symmetry – the area in the tails equidistant from the mean will be equal.
(ii) The answers to a, b, c sum to one because the events cover the entire area under the normal curve which by definition, must sum to 1.

NCT 6.28

$P(Z > 1.5) = 1 - F_z(1.5) = .0668$

NCT 6.34

a. $P(Z > -1.28) = .9$, $-1.28 = \frac{Xi-150}{40}$, $Xi = 98.8$

b. $P(Z < .84) = .8$, $.84 = \frac{Xi-150}{40}$, $Xi = 183.6$

c. $P(X \geq 1) = 1 – P(X = 0) = 1-[P(Z<\frac{120-150}{40})]^2 = 1 – [P(Z < -.75)]^2 = 1 – (.2266)^2$
$= .9487$

NCT 6.56

$P(X > 18) = e^{-(18/15)} = .3012$

NCT 4.68

Let *A* – Regularly read business section, *B* – Occasionally, *C* – Never, TS – Traded stock

a. $P(C) = .25$

b. $P(\text{TS}) = .32$

c. $P(\text{TS}|C) = P(\text{TS}\cap C)/P(C) = .04/.25 = .16$

d. $PC|\text{TS}) = P(\text{TS}\cap C/ P(\text{TS}) = .04/.32 = .125$

e. $P(\text{TS}|\bar{A}) = P(\text{TS}\cap(B\cup C)/PB\cup C) = (.10 + .04)/(.41+.25) = .2121$

NCT 4.72

Let R – Readers, V – voted in the last election

a. $P(\text{V}) = .76$

b. $P(\text{R}) = .77$

c. $P(\bar{V}\,|\,\bar{R}) = P(\bar{V} \cap \bar{R})/P(\bar{R}) = .1/.23 = .4348$

NCT 4.75

P(HC) = .42, P(WS) = .22, P(WS|HC) = .34

a. $P(\text{HC} \cap \text{WS}) = P(\text{WS}|\text{HC})P(\text{HC}) = (.42)(.34) = .1428$

b. $P(\text{HC} \cup \text{WS}) = P(\text{HC}) + P(\text{WS}) – P(\text{HC} \cap \text{WS}) = .42 + .22 - .1428 = .4972$

c. $P(\text{HC}|\text{WS}) = P(\text{HC} \cap \text{WS})/P(\text{WS}) = .1428 / .22 = .6491$

NCT 4.78

Let M – faulty machine, I – impurity

$P(\text{M}) = .4$, $P(\text{I}|\text{M}) = .1$, $P(\text{I}) = P(\text{I} \cap \text{M}) + P(\text{I} \cap \bar{M}) = (.4)(.1) + 0 = .04$ $P(\text{M}|\bar{I}) =$
$P(\bar{I} \cap \text{M}) / P(\bar{I}) = [P(\text{M}) – P(\text{I}\bar{I}\,\text{M})] / P(\bar{I}) = (.4 - .04)/.96 = .375$

11. Let X be time of a visit to a nurse.

Then it is given that $X : \exp(\frac{1}{6})$

(i) $P(X > 10) = 1 - P(X \le 10) = 1 - (1 - e^{-\frac{1}{6}\cdot 10}) = 1 - (1 - e^{-\frac{5}{3}}) = 1 - (1 - 0.189) = 0.189$

(ii) $P(3 \le X \le 9) = P(3 < X \le 9) = P(X \le 9) - P(X \le 3) =$

$= 1 - e^{-\frac{1}{6}\cdot 9} - [1 - e^{-\frac{1}{6}\cdot 3}] = -0.223 + 0.607 = 0.384$