

Problems for Class 6

TRUE or FALSE problems

State whether you believe the given statement is TRUE or FALSE and provide a brief argument for your answer.

1. The joint pmf of random variables X and Y, $f(X, Y)$, is given by:

$$f(X, Y) = \begin{cases} = 0.09 & (X, Y) = (0, 0) \\ = 0.07 & (X, Y) = (1, 0) \\ = 0.38 & \text{for } (X, Y) = (0, 1) \\ = 0.11 & (X, Y) = (1, 1) \\ = 0.01 & (X, Y) = (2, 1) \\ = 0 & \textit{otherwise} \end{cases}$$

2. The joint pmf of random variables X and Y, $f(X, Y)$, is given by:

$$f(X, Y) = \begin{cases} = 0.31 & (X, Y) = (11, 32) \\ = 0.12 & (X, Y) = (11, 47) \\ = 0.09 & \text{for } (X, Y) = (35, 32) \\ = 0.02 & (X, Y) = (56, 32) \\ = 0.46 & (X, Y) = (72, 52) \\ = 0 & \textit{otherwise} \end{cases}$$

Exercises

Exercises 1-3: NCT 5.82 (a), (b), (c); 5.84 (a) (b); 5.85 (a), (b)

4. For the example presented in lectures 11 and 12, find:

- The expected value of X.
- The expected value of Y.
- The variance of X.
- The variance of Y.
- Find the expected value for each distinct conditional distribution of X, conditional on Y.

- (f) Find the expected value for each distinct conditional distribution of Y , conditional on X .
- (g) Let $Z = g(X, Y) = (X+Y)/2$. Find $E(Z)$.

5. [final exam 05-06] In the country of Bathland, university graduates are paid more than non-graduates. In that country the amount of 100 zgups is a substantial sum of money. The joint probability table for the population by being a university graduate and earning more than 100 zgups annually is given below.

	100 or more	Less than 100
Graduate	0.21	0.11
Non-graduate	0.23	0.45

Let the random variable X be such that:

- $X = 0$ if a person is not a university graduate
 $X = 1$ if a person is a university graduate

And the random variable Y :

- $Y = 0$ if a person earns less than 100 zgups
 $Y = 1$ if a person earns at least 100 zgups

- (i) Specify the joint pmf of X and Y .
(ii) Specify the marginal pmfs of X and Y .
(iii) Find all the distinct conditional pmf's of Y , conditional upon X .
(iv) Let $Z = XY$. Find $E(Z)$.

6. [resit exam 05-06] In the country of Slumberland the joint probability table for the population employment by gender is given by the following table:

	Employed	Not employed
Men	0.21	0.29
Women	0.18	0.32

Let the random variable X be such that:

- $X = 0$ if a person is employed
 $X = 1$ if a person is not employed

And the random variable Y :

- $Y = 0$ for men
 $Y = 1$ for women

- (i) Specify the joint pmf of X and Y .

- (ii) Specify the marginal pmfs of X and Y .
- (iii) Find all the distinct conditional pmf's of Y , conditional upon X .
- (iv) Let $Z = XY$. Find $E(Z)$.