University of Bath
Department of Economics and International Development
EC10003 Introduction to Statistics
Fall 2006
Thanos Mergoupis

Answers to Problems for Class 6

TRUE or FALSE problems

1. FALSE
The sum of the *f(X,Y)* is less than 1.

2. TRUE
It satisfies the conditions for a joint pmf.

Exercises

NCT 5.82

a. $f_X(0) = .07 + .07 + .06 + .02 = .22$
$f_X(1) = .09 + .06 + .07 + .04 = .26$
$f_X(2) = .06 + .07 + .14 + .16 = .43$
$f_X(3) = .01 + .01 + .03 + .04 = .09$
f_X=0 otherwise
$\mu_x = 0 + .26 + 2(.43) + 3(.09) = 1.39$

b. $f_Y(0) = .07 + .09 + .06 + .01 = .23$
$f_Y(1) = .07 + .06 + .07 + .01 = .21$
$f_Y(2) = .06 + .07 + .14 + .03 = .30$
$f_Y(3) = .02 + .04 + .16 + .40 = .26$
f_Y=0 otherwise
$\mu_y = 0 + .21 + 2(.3) + 3(.26) = 1.59$

c. $g_{Y|X}(0|3) = .01/.09 = .1111$
$g_{Y|X}(1|3) = .01/.09 = .1111$
$g_{Y|X}(2|3) = .03/.09 = .3333$
$g_{Y|X}(3|3) = .04/.09 = .4444$
$g_{Y|X}$=0 otherwise

NCT 5.84

a. $f_Y(0) = .08 + .03 + .01 = .12$
$f_Y(1) = .13 + .08 + .03 = .24$
$f_Y(2) = .09 + .08 + .06 = .23$
$f_Y(3) = .06 + .09 + .08 = .23$
$f_Y(4) = .03 + .07 + .08 = .18$

b.

$$g_{Y|X}(Y \mid X=3) = \begin{cases} =1/26 & Y=0 \\ =3/26 & Y=1 \\ =6/26 & Y=2 \\ =8/26 \quad \text{for} & Y=3 \\ =8/26 & Y=4 \\ =0 & \textit{otherwise} \end{cases}$$

NCT 5.85

a.

$$f(X,Y) = \begin{cases} =0.54 & (X,Y)=(0,0) \\ =0.30 & (X,Y)=(0,1) \\ =0.01 \quad \text{for} & (X,Y)=(1,0) \\ =0.15 & (X,Y)=(1,1) \\ =0 & \textit{otherwise} \end{cases}$$

b.

$$g_{Y|X}(Y \mid X=1) = \begin{cases} =1/16 & Y=0 \\ =15/16 \quad \text{for} & Y=1 \\ =0 & \textit{otherwise} \end{cases}$$

4. See answers to exercises 5 and 6 below.

5. First we convert the given table to table of joint probabilities and marginal probabilities relating to the values of the two random variables. We have:

	X			
Y		0	1	
	0	0.45	0.23	0.68
	1	0.11	0.21	0.32
		0.56	0.44	1.00

(i) The joint pmf of X and Y *f(X,Y)* is then given by:

$$f(X,Y)=\begin{cases}=0.45 & (X,Y)=(0,0)\\ =0.23 & (X,Y)=(0,1)\\ =0.11 \quad \text{for} & (X,Y)=(1,0)\\ =0.21 & (X,Y)=(1,1)\\ =0 & \textit{otherwise}\end{cases}$$

(ii) The marginal pmf of X, $f_1(X)$ is given by:

$$f_1(X)=\begin{cases}=0.68 & X=0\\ =0.32 \quad \text{for} & X=1\\ =0 & \textit{otherwise}\end{cases}$$

The marginal pmf of Y, $f_2(Y)$ is given by:

$$f_2(Y)=\begin{cases}=0.56 & Y=0\\ =0.32 \quad \text{for} & Y=1\\ =0 & \textit{otherwise}\end{cases}$$

(iii) For $g_1(Y \mid X=0)$ we have:

$$g_2(Y=0 \mid X=0)=\frac{f(0,0)}{f_1(0)}=\frac{0.45}{0.68}=0.66$$

$$g_2(Y=1 \mid X=0)=\frac{f(0,1)}{f_1(0)}=\frac{0.23}{0.68}=0.34$$

So:

$$g_2(Y \mid X=0)=\begin{cases}=0.66 & Y=0\\ =0.34 \quad \text{for} & Y=1\\ =0 & \textit{otherwise}\end{cases}$$

For $g_2(Y \mid X=1)$ we have:

$$g_2(Y=0 \mid X=1)=\frac{f(1,0)}{f_1(1)}=\frac{0.11}{0.32}=0.34$$

$$g_2(Y=1 \mid X=1) = \frac{f(1,1)}{f_1(1)} = \frac{0.21}{0.32} = 0.66$$

So:

$$g_2(Y \mid X=1) = \begin{cases} = 0.34 & Y=0 \\ = 0.66 \quad \text{for} & Y=1 \\ = 0 & \textit{otherwise} \end{cases}$$

(iv)

$$E(Z) = E(XY) = \sum_i \sum_j x_i y_j f(x_i, y_j) =$$

$$= 0 \cdot 0 \cdot f(0,0) + 1 \cdot 0 \cdot f(1,0) + 0 \cdot 1 \cdot f(0,1) + 1 \cdot 1 \cdot f(1,1) = 0.21$$

6. First we convert the given table to table of joint probabilities and marginal probabilities relating to the values of the two random variables. We have:

		X		
Y		0	1	
	0	0.21	0.29	0.50
	1	0.18	0.32	0.50
		0.39	0.61	1.00

(i) The joint pmf of X and Y *f(X,Y)* is then given by:

$$f(X,Y) = \begin{cases} = 0.21 & (X,Y) = (0,0) \\ = 0.18 & (X,Y) = (0,1) \\ = 0.29 \quad \text{for} & (X,Y) = (1,0) \\ = 0.32 & (X,Y) = (1,1) \\ = 0 & \textit{otherwise} \end{cases}$$

(ii) The marginal pmf of X, $f_1(X)$ is given by:

$$f_1(X) = \begin{cases} = 0.39 & X=0 \\ = 0.61 \quad \text{for} & X=1 \\ = 0 & \textit{otherwise} \end{cases}$$

The marginal pmf of Y, $f_2(Y)$ is given by:

$$f_2(Y) = \begin{cases} = 0.50 & Y = 0 \\ = 0.50 \quad \text{for} & Y = 1 \\ = 0 & \textit{otherwise} \end{cases}$$

(iii) For $g_1(Y \mid X = 0)$ we have:

$$g_2(Y = 0 \mid X = 0) = \frac{f(0,0)}{f_1(0)} = \frac{0.21}{0.39} = 0.54$$

$$g_2(Y = 1 \mid X = 0) = \frac{f(0,1)}{f_1(0)} = \frac{0.18}{0.39} = 0.46$$

So:

$$g_2(Y \mid X = 0) = \begin{cases} = 0.54 & Y = 0 \\ = 0.46 \quad \text{for} & Y = 1 \\ = 0 & \textit{otherwise} \end{cases}$$

For $g_2(Y \mid X = 1)$ we have:

$$g_2(Y = 0 \mid X = 1) = \frac{f(1,0)}{f_1(1)} = \frac{0.29}{0.61} = 0.475$$

$$g_2(Y = 1 \mid X = 1) = \frac{f(1,1)}{f_1(1)} = \frac{0.32}{0.61} = 0.525$$

So:

$$g_2(Y \mid X = 1) = \begin{cases} = 0.475 & Y = 0 \\ = 0.525 \quad \text{for} & Y = 1 \\ = 0 & \textit{otherwise} \end{cases}$$

(iv)

$$E(Z) = E(XY) = \sum_i \sum_j x_i y_j f(x_i, y_j) =$$

$$= 0 \cdot 0 \cdot f(0,0) + 1 \cdot 0 \cdot f(1,0) + 0 \cdot 1 \cdot f(0,1) + 1 \cdot 1 \cdot f(1,1) = 0.32$$