University of Bath
Department of Economics and International Development
EC10003 Introduction to Statistics
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Problems for Class 7

TRUE or FALSE problems

1. FALSE
The conditional pmfs of X conditional upon Y are function of X.

2. FALSE
The conditional expectation function of Y conditional upon X is a function of X

3. FALSE
Only the reverse is true.

4. FALSE
The variance gets smaller.

5. FALSE
The sample mean is a sample statistic because its value is a function of the random sample.

6. FALSE
The mean of the sample from population B can be larger than the mean of the sample from population A, depending on the actual samples drawn.
The stated relationship holds for the expected values of the sample means.

7. FALSE
The mean is 5, and the variance is 3/5.

8. TRUE
By the sample mean theorem, since $E(X) = 3$.

Exercises

NCT 5.82
(d). E(XY) = 0 + 1(3)(.14) + 1(4)(.23) + 1(5)(.10) + 2(3)(.07) + 2(4)(.16) + 2(5)(.11) = 4.64

$\mu_x = 0 + .47 + 2(.34) = 1.15$

$\mu_y = 3(.3) + 4(.46) + 5(.24) = 3.94$

$Cov(X,Y) = 4.64 - (1.15)(3.94) = .109$

The covariance indicates that there is a positive association between the number of lines in the advertisement and the volume of inquiries.

e. No, because $Cov(X,Y) \neq 0$

NCT 5.84
(c) No, because $f(X=3,Y=4) = .08 \neq .0468 = f_1(X=3)f_2(Y=4)$

NCT 5.85
(c) E(XY) = .15
$\mu_x = 0 + 1(.16) = .16$
$\mu_y = 0 + 1(.45) = .45$
$Cov(X,Y) = .15 - (.16)(.45) = .078$
The covariance indicates that there is a positive association between brand watchers of a late-night talk show and brand name recognition.

4. See answers to 5 and 6 below for method of solution.

5. (a) We have:

$$E(Y \mid X=0) = \sum_j y_j g_2(y_j \mid X=0) = 0 \cdot 0.66 + 1 \cdot 0.34 = 0.34$$
$$E(Y \mid X=1) = \sum_j y_j g_2(y_j \mid X=1) = 0 \cdot 0.34 + 1 \cdot 0.66 = 0.66$$

So the CEF E(Y|X) is:

$$E(Y \mid X) = \begin{cases} 0.34 & \text{for} \quad X=0 \\ 0.66 & \quad\quad\;\; X=1 \end{cases}$$

(b) For the covariance we need E(X) and E(Y).

$$E(X) = \sum_i x_i f_1(x_i) = 0 \cdot 0.68 + 1 \cdot 0.32 = 0.32 = \mu_X$$

$$E(Y) = \sum_j y_j f_2(y_j) = 0 \cdot 0.56 + 1 \cdot 0.44 = 0.44 = \mu_Y$$

We then have:

$$Cov(X,Y) = \sum_i \sum_j (x_i - \mu_X)(y_j - \mu_Y) f(x_i, y_j) =$$
$$= (0-0.32)(0-0.44) f(0,0) + (1-0.32)(0-0.44) f(1,0) +$$
$$+ (0-0.32)(1-0.44) f(0,1) + (1-0.32)(1-0.44) f(1,1) =$$
$$= 0.0634 - 0.0412 - 0.0329 + 0.0800 = 0.0693$$

6. (a) We have:

$$E(Y \mid X=0)=\sum_j y_j g_2(y_j \mid X=0)=0\cdot 0.54+1\cdot 0.46=0.46$$
$$E(Y \mid X=1)=\sum_j y_j g_2(y_j \mid X=1)=0\cdot 0.475+1\cdot 0.525=0.525$$

So the CEF E(Y|X) is:

$$E(Y \mid X)=\begin{cases}0.46 \\ 0.525\end{cases} \quad \text{for} \quad \begin{matrix} X=0 \\ X=1 \end{matrix}$$

(b) For the covariance we need E(X) and E(Y).

$$E(X)=\sum_i x_i f_1(x_i)=0\cdot 0.39+1\cdot 0.61=0.61=\mu_X$$

$$E(Y)=\sum_j y_j f_2(y_j)=0\cdot 0.50+1\cdot 0.50=0.50=\mu_Y$$

We then have:

$$Cov(X,Y)=\sum_i\sum_j (x_i-\mu_X)(y_j-\mu_Y)f(x_i,y_j)=$$
$$=(0-0.61)(0-0.50)f(0,0)+(1-0.61)(0-0.50)f(1,0)+$$
$$+(0-0.61)(1-0.50)f(0,1)+(1-0.61)(1-0.50)f(1,1)=0.015$$