University of Bath
Department of Economics and International Development
EC10003 Introduction to Statistics
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Thanos Mergoupis

# Problems for Class 8

## TRUE or FALSE problems

State whether you believe the given statement is TRUE or FALSE and provide a brief argument for your answer.

1. When sampling from a normal population, the sample mean follows a standard normal distribution.

2. The Mean Squared Error of the sample mean as an estimator of the population mean is always equal to the variance of the sample mean.

3. The Mean Squared Error of the sample mean as an estimator of the population mean is always no greater than the population variance.

## Exercises

Exercises 1-6: NCT 7.7 (a),(b),(c),(d); 7.9; 7.11; 7.15; 7.19; 8.6.

7. [05-06 final] Define the random variable X as follows:

X = 1 if when rolling a pair of fair six-sided dice we obtain at least one 3 (i.e. at least one die must be a 3).

X = 0 for any other roll.

i. Define the pmf of X.

Suppose that you roll the dice twice, hence obtaining a sample of size 2 from this population.

ii. Derive the sampling distribution of the sample mean.
iii. Verify that the sample mean is an unbiased estimator of the population mean.
iv. What is the probability that the sample mean is between 0.10 and 0.40?

8. We have the r.v. X with pmf:

$f(1) = 3/4$

$f(0) = 1/4$

*f(x)* = 0 otherwise

So the values of X can be thought to be determined by the flip of loaded coin, which gives, say, heads (H) 3/4 of the time, and tails (T) 1/4 of the time. Then X=1 if H, and X=0 if T.

(a) Consider sampling of size 2 from this population. Derive the sampling distribution of the sample mean and the sample variance.
(b) Find the mean and variance of the sample mean in (a) and verify the sample mean theorem.
(c) Consider sampling of size 3 from this population. Derive the sampling distribution of the sample mean.
(d) Standardise the sample means in (a) and (c) above and find P( $Z_2 \leq 0.1$) and P( $Z_3 \leq 0.1$), where $Z_2$ is the standardised sample mean of the sample size 2 and $Z_3$ is the standardised sample mean of the sample size 3. Find the magnitude of your error if you had used the standard normal to calculate these probabilities instead. Do your results support the CLT?
(e) Determine whether the sample variance is an unbiased estimator of the population variance.
(f) Consider the following sample statistic:

$$S_*^2 = \frac{1}{n}\sum_i (X_i - \bar{X})^2$$

For sampling size 2, determine whether it is an unbiased estimator of the population variance.
(g) For sampling size 2, which estimator of the sample variance has the lowest variance, the sample variance or $S_*^2$ ?

Consider again random sampling size 3 from this population.
(h) Determine whether the sample midrange is an unbiased estimator of the population mean.
(i) Compare the MSE of the sample midrange and the sample mean as estimators of the population mean.

9. [05-06 final] For year 2003, the logarithm of annual individual earnings followed a normal distribution with mean 9.711 and variance 75.682 (all logarithms are taken to be natural logarithms). For year 2002, the logarithm of annual individual earnings followed a normal distribution with mean 9.702 and variance 75.093. The correlation between the two log earnings was 0.73. Let Y denote an individual's average log earnings over the two years.
(i) If you randomly draw one individual, what is the probability that Y is more than 10.223?
(ii) If you draw a random sample of 3 individuals, what is the probability that the sample mean of Y is more than 10.223?