University of Bath
Department of Economics and International Development
EC10003 Introduction to Statistics
Fall 2006
Thanos Mergoupis

Answers to Problems for Class 8

TRUE or FALSE problems

State whether you believe the given statement is TRUE or FALSE and provide a brief argument for your answer.

1. FALSE
The standardised sample mean follows a standard normal distribution.

2. TRUE
Direct from the definition of MSE.

3. TRUE
The MSE of the sample mean is equal to the variance of the sample mean (because unbiased), which in turn is no greater than the population variance (equal only when sample size is n=1) by the sample mean theorem.

Exercises

NCT 7.7

a. mean and variance of the sampling distribution for the sample mean

$$\mu_{\bar{x}} = \mu = 200$$

$$\sigma^2_{\bar{x}} = \sigma^2 / n = 625/25 = 25 \quad \sigma_{\bar{x}} = \sqrt{\sigma_{\bar{x}}^{\,2}} = \sqrt{25}$$

b. Probability that $\bar{x} > 209$ $\quad z_{\bar{x}} = \dfrac{209-200}{\sqrt{25}} = 1.80$ 1 – Fz(1.80) = .0359

c. Probability that $198 \le \bar{x} \le 211$ $\quad z_{\bar{x}} = \dfrac{198-200}{\sqrt{25}} = -.40$ 1 – Fz(.40) = 0.3446

$z_{\bar{x}} = \dfrac{211-200}{\sqrt{25}} = 2.20$ Fz(2.20) = .9861. .9861 - .3446 = .6415

d. Probability that $\bar{x} \le 202$ $\quad z_{\bar{x}} = \dfrac{202-200}{\sqrt{25}} = .40$ Fz = .6554

NCT 7.9

a. E($\bar{X}$) = $\mu_{\bar{x}}$ = 92.

b. $\sigma_{\bar{x}}^{\,2} = \dfrac{\sigma^2}{n} = \dfrac{(3.6)^2}{4} = 3.24$

c. $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{3.6}{2} = 1.8$

d. P(Z > $\dfrac{93-92}{1.8}$) = P(Z > .56) = .2877

NCT 7.11

a. i) P(Z > $\frac{24-25}{2}$) = P(Z < -.5) = .3085.

ii) P(Z < $\frac{24-25}{2/\sqrt{4}}$) = P(Z < -1) =.1587

iii) P(Z < $\frac{24-25}{2/\sqrt{16}}$) = P(Z < -2) = .0228

b. As the sample size increases, the standard error of the sampling distribution will decrease. That is, as the sample size increases, the sampling distribution of the sample means will clump up tighter around the true population mean. The graph would show a tighter distribution with less area in the tails.

NCT 7.15

a. $\sigma_{\bar{x}} = \frac{.6}{\sqrt{4}} = .3$

b. P(Z < $\frac{19.7-20}{.3}$) = P(Z < -1) = .1587

c. P(Z > $\frac{20.6-20}{.3}$) = P(Z > 2) = .0228

d. P($\frac{19.5-20}{.3}$ < Z < $\frac{20.5-20}{.3}$) = P(-1.67 < Z < 1.67) = .905

e. P($\frac{19.5-20}{.6/\sqrt{2}}$ < Z < $\frac{20.5-20}{.6/\sqrt{2}}$) = P(-1.18 < Z < 1.18) = .762

NCT 7.19

a. P(Z > 1.645) = .10, 1.645 = $\frac{1}{3.8/\sqrt{n}}$, n = 39.075, take n = 40

b. larger

c. larger

NCT 8.6

a. $E(\bar{X}) = \frac{1}{2}E(X_1) + \frac{1}{2}E(X_2) = \frac{\mu}{2} + \frac{\mu}{2} = \mu$

$E(Y) = \frac{1}{4}E(X_1) + \frac{3}{4}E(X_2) = \frac{\mu}{4} + \frac{3\mu}{4} = \mu$

$E(Z) = \frac{1}{3}E(X_1) + \frac{2}{3}E(X_2) = \frac{\mu}{3} + \frac{2\mu}{3} = \mu$

b. $Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{1}{4}Var(X_1) + \frac{1}{4}Var(X_2) = \frac{1}{2}\frac{\sigma^2}{8} = \frac{\sigma^2}{4}$

$$Var(Y) = \frac{1}{16}Var(X_1) + \frac{9}{16}Var(X_2) = \frac{5\sigma^2}{8}$$
$$Var(Z) = \frac{1}{9}Var(X_1) + \frac{4}{9}Var(X_2) = \frac{5\sigma^2}{9}$$
$\bar{X}$ is most efficient since $Var(\bar{X}) < Var(Y) < Var(Z)$

c. Relative efficiency between Y and $\bar{X}$: $\frac{Var(Y)}{Var(\bar{X})} = \frac{5}{2} = 2.5$

Relative efficiency between Z and $\bar{X}$: $\frac{Var(Z)}{Var(\bar{X})} = \frac{20}{9} = 2.222$

7. (a) (i)

P(X=1) = P(at least one 3 in a roll of two dice) = P[(3,1), (1,3), (3,2), (2,3), (3,3), (4,3), (3,4), (5,3), (3,5), (6,3), (3,6)] = 11 x (1/36) = 11/36

P(X=0) = 1- P(X=1) = 1 – (11/36) = 25/36

Let f(X) be the pmf of X. Using the probability assignment rule for discrete random variables:

f(X=1) = P(X=1) = 11/36

f(X=0) = P(X=0) = 25/36

Then:

$$f(X) = \begin{cases} = \frac{11}{36} & X = 1 \\ = \frac{26}{36} & \text{for } X = 0 \\ = 0 & otherwise \end{cases}$$

For (ii) and (iii), follow methods of (a) and (b) in exercise 8.

8. (a) We have random sampling size 2 from the r.v. X with pmf:

f(X=1) = 3/4
f(X=0) = 1/4
f(X) = 0 otherwise

With random sampling size 2 we have the following possible samples:

(1,1), (1,0), (0,1), (0,0)

The sample (1,1) gives

$\overline{X} = 1$, and

$$S^2 = \frac{1}{2-1}\sum_i (X_i - \overline{X})^2 = 1[(1-1)^2 + (1-1)^2] = 0$$

The sample (1,0) gives:

$$\overline{X} = \frac{1}{2}$$

$$S^2 = \frac{1}{2-1}\sum_i (X_i - \overline{X})^2 = 1[(1-\frac{1}{2})^2 + (0-\frac{1}{2})^2] = \frac{1}{2}$$

The sample (0,1) gives:

$$\overline{X} = \frac{1}{2}$$

$$S^2 = 1[(0-\frac{1}{2})^2 + (1-\frac{1}{2})^2] = \frac{1}{2}$$

And the sample (0,0) gives:

$$\overline{X} = 0$$

$$S^2 = 1[(0-0)^2 + (0-0)^2] = 0$$

Since:

$$P[(1,1)] = \frac{9}{16}$$

$$P[(1,0)] = \frac{3}{16}$$

$$P[(0,1)] = \frac{3}{16}$$

$$P[(0,0)] = \frac{1}{16}$$

For the sample mean we have:

$$P(\overline{X} = 0) = P[(0,0)] = \frac{1}{16}$$

$$P(\overline{X} = \frac{1}{2}) = P[(1,0) \text{ or } (0,1)] = P[(1,0)] + P[(0,1)] = \frac{3}{16} + \frac{3}{16} = \frac{3}{8}$$

$$P(\bar{X}=1)=P[(1,1)]=\frac{9}{16}$$

Therefore the sampling distribution $f_{\bar{X}}(\bar{X})$ of the sample mean is:

$$f_{\bar{X}}(\bar{X})=\begin{cases} =\frac{1}{16} & \bar{X}=0 \\ =\frac{6}{16} \quad \text{for} & \bar{X}=\frac{1}{2} \\ =\frac{9}{16} & \bar{X}=1 \\ =0 & \textit{otherwise} \end{cases}$$

For the sample variance we have:

$$P(S^2=0)=P[(1,1)\ \text{or}\ (0,0)]=P[(1,1)]+P[(0,0)]=\frac{9}{16}+\frac{1}{16}=\frac{10}{16}=\frac{5}{8}$$

$$P(S^2=\frac{1}{2})=P[(1,0)\ \text{or}\ (0,1)]=P[(1,0)]+P[(0,1)]=\frac{3}{16}+\frac{3}{16}=\frac{3}{8}$$

Let the pmf of S^2 be $h(S^2)$. Then:

$$h(S^2)=\frac{5}{8} \qquad \text{for } S^2=0$$
$$h(S^2)=\frac{3}{8} \qquad \text{for } S^2=\frac{1}{2}$$
$$h(S^2)=0 \ \text{o/w}$$

(b) The population mean μ_x is:

$$\mu_x=E(X)=\sum_i x_i f(x_i)=0\cdot\frac{1}{4}+1\cdot\frac{3}{4}=\frac{3}{4}$$

The population variance σ^2 is:

$$\sigma^2=E[(X-\mu_X)^2]=\sum_i (x_i-\mu_X)^2 f(x_i)=(1-\frac{3}{4})^2\cdot\frac{3}{4}+(0-\frac{3}{4})^2\cdot\frac{1}{4}=$$

$$=\frac{1}{16}\cdot\frac{3}{4}+\frac{9}{16}\cdot\frac{1}{4}=\frac{12}{64}=\frac{3}{16}$$

From (a) we have:

$$E(\bar{X}) = \sum_i \bar{x}_i \cdot f_{\bar{X}}(\bar{x}_i) = 0 \cdot \frac{1}{16} + \frac{1}{2} \cdot \frac{6}{16} + 1 \cdot \frac{9}{16} = \frac{12}{16} = \frac{3}{4} = \mu_X$$

as predicted by the sample mean theorem. And:

$$V(\bar{X}) = \sum_i (\bar{x}_i - E(\bar{X}))^2 \cdot f_{\bar{X}}(\bar{x}_i) =$$

$$= (0 - \frac{3}{4})^2 \cdot \frac{1}{16} + (\frac{1}{2} - \frac{3}{4})^2 \cdot \frac{6}{16} + (1 - \frac{3}{4})^2 \cdot \frac{9}{16} = \frac{9}{16} \cdot \frac{1}{16} + \frac{1}{16} \cdot \frac{6}{16} + \frac{1}{16} \cdot \frac{9}{16} =$$

$$= \frac{9}{256} + \frac{6}{256} + \frac{9}{256} = \frac{24}{256} = \frac{3}{32} = \frac{\sigma^2}{n}$$

as predicted by the sample mean theorem.

(c) With random sampling size 3 we have $2^3 = 8$ possible samples:

(1,1,1), (1,1,0), (1,0,1), (0,1,1), (1,0,0), (0,1,0), (0,0,1), (0,0,0)

The sample (1,1,1) has probability:

$$P\left[(1,1,1)\right] = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$$

and gives a sample mean of $\bar{X} = 1$.

The samples (1,1,0), (1,0,1), and (0,1,1), have probability:

$$P\left[(1,1,0)\right] = P\left[(1,0,1)\right] = P\left[(0,1,1)\right] = \frac{9}{64}$$

and give a sample mean of $\bar{X} = \frac{2}{3}$.

The samples (1,0,0), (0,1,0), and (0,0,1), have probability:

$$P\left[(1,0,0)\right] = P\left[(0,1,0)\right] = P\left[(0,0,1)\right] = \frac{3}{64}$$

and give a sample mean of $\bar{X} = \frac{1}{3}$.

The sample (0,0,0), has probability:

$$P\left[(0,0,0)\right]=\frac{1}{64}$$

and gives a sample mean of $\bar{X}=0$.

The sampling distribution of the sample mean then is:

$$f_{\bar{X}}(\bar{X})=\begin{cases}=\frac{1}{64} & \bar{X}=0\\ =\frac{3}{64} & \bar{X}=\frac{1}{3}\\ =\frac{9}{64} & \text{for } \bar{X}=\frac{2}{3}\\ =\frac{27}{64} & \bar{X}=1\\ =0 & \textit{otherwise}\end{cases}$$

(d) To standardise the sample means in (a) we need to subtract their mean and divide by their standard deviation (which are given by the sample mean theorem). We therefore have:

$$Z_2=\frac{\bar{X}_2-\mu_X}{\sigma/\sqrt{n}}=\frac{\bar{X}_2-\frac{3}{4}}{\sqrt{3/32}}$$

and

$$Z_3=\frac{\bar{X}_3-\mu_X}{\sigma/\sqrt{n}}=\frac{\bar{X}_2-\frac{3}{4}}{\sqrt{1/16}}=\frac{\bar{X}_2-\frac{3}{4}}{1/4}$$

To evaluate the required probabilities we have:

$$\text{P}(\text{Z}_2\le 0.1)\text{=P}\left(\frac{\bar{X}_2-\frac{3}{4}}{\sqrt{3/32}}\le 0.1\right)=\text{P}(\bar{X}_2-\frac{3}{4}\le 0.1\cdot\sqrt{3/32})=\text{P}(\bar{X}_2\le\frac{\sqrt{6}}{80}+\frac{3}{4})=$$

$$=\text{P}(\bar{X}_2\le\frac{\sqrt{6}+60}{80})=\text{P}(\bar{X}_2\le 0.78)=\frac{7}{16}=0.4375$$

As can be verified by the cdf of $\bar{X}_2$ which can be derived from the pmf in (a).

$$P(Z_3 \le 0.1) = P\left(\frac{\bar{X}_3 - \frac{3}{4}}{1/4} \le 0.1\right) = P\left(\bar{X}_3 - \frac{3}{4} \le \frac{1}{40}\right) = P\left(\bar{X}_3 \le \frac{1}{40} + \frac{3}{4}\right) =$$

$$= P\left(\bar{X}_3 \le \frac{31}{40}\right) = P(\bar{X}_3 \le 0.775) = \frac{37}{64} = 0.5781$$

As can be verified by the cdf of $\bar{X}_3$ which can be derived from the pmf in (c).

Now we need to evaluate the corresponding approximate probabilities as approximated by the CLT.

According to the CLT:

$$P(Z_2 \le 0.1) \approx \Phi(0.1) = 0.5398$$

and

$$P(Z_3 \le 0.1) \approx \Phi(0.1) = 0.5398$$

The approximation error is equal to the distance between the approximate value and the exact value, i.e.

$$\text{Approximation error} = \left|\Phi(0.1) - P(Z_n \le 0.1)\right|$$

For n=2 we have:

$$\text{Approximation error} = \left|\Phi(0.1) - P(Z_2 \le 0.1)\right| = \left|0.5398 - 0.4375\right| = 0.1023$$

For n=3 we have:

$$\text{Approximation error} = \left|\Phi(0.1) - P(Z_3 \le 0.1)\right| = \left|0.5398 - 0.5781\right| = 0.0383$$

Which both confirms and shows how powerful the CLT is, as the approximation error is small and is reduced drastically just going from sample size 2 to sample size 3.

(e) Using the results of (a) and (b) we have:

$$E(S^2) = \sum_i S_i^2 \cdot h(S_i^2) = 0 \cdot \frac{5}{8} + \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{16} = \sigma^2$$

(f) For $S_*^2 = \frac{1}{n}\sum_i (x_i - \bar{x})^2$, the sample (1,1) gives:

$$S_*^2 = \frac{1}{2}\sum_i (x_i - \overline{X})^2 = \frac{1}{2}[(1-1)^2 + (1-1)^2] = 0$$

The sample (1,0) gives:

$$S_*^2 = \frac{1}{2}\sum_i (x_i - \overline{X})^2 = \frac{1}{2}[(1-\frac{1}{2})^2 + (0-\frac{1}{2})^2] = \frac{1}{2}(\frac{1}{4}+\frac{1}{4}) = \frac{1}{4}$$

The sample (0,1) gives:

$$S_*^2 = \frac{1}{2}[(0-\frac{1}{2})^2 + (1-\frac{1}{2})^2] = \frac{1}{2}(\frac{1}{4}+\frac{1}{4}) = \frac{1}{4}$$

And the sample (0,0) gives:

$$S_i^2 = \frac{1}{2}[(0-0)^2 + (0-0)^2] = 0$$

So the pmf of S_*^2 is:

$$h_*(S_*^2) = \frac{5}{8} \quad \text{for } S_*^2 = 0$$
$$h_*(S_*^2) = \frac{3}{8} \quad \text{for } S_*^2 = \frac{1}{4}$$

$$h_*(S_*^2) = 0 \quad \text{o/w}$$

Then:

$$E(S_*^2) = \sum_i S_{*i}^2 \cdot h_*(S_{*i}^2) = 0 \cdot \frac{5}{8} + \frac{1}{4}\cdot\frac{3}{8} = \frac{3}{32} \neq \sigma^2$$

(g) $Var(S^2) = E[(S^2 - E(S^2))^2] = \sum_i [S_i^2 - E(S^2)]^2 \cdot h(S_i^2) =$

$$= (0-\frac{3}{16})^2 \cdot \frac{5}{8} + (\frac{1}{2}-\frac{3}{16})^2 \cdot \frac{3}{8} = \frac{13^2}{16^2}\cdot\frac{5}{8} + \frac{5^2}{16^2}\cdot\frac{3}{8} =$$

$$= \frac{845+75}{2{,}048} = \frac{920}{2{,}048} = \frac{115}{256}$$

$$Var(S_*^2) = \sum_i [S_{*i}^2 - E(S_*^2)]^2 \cdot h_*(S_{*i}^{\ 2}) =$$

$$= (0 - \frac{3}{32})^2 \cdot \frac{5}{8} + (\frac{1}{4} - \frac{3}{32})^2 \cdot \frac{3}{8} = \frac{29^2}{32^2} \cdot \frac{5}{8} + \frac{5^2}{32^2} \cdot \frac{3}{8} =$$

$$= \frac{4{,}205 + 75}{8{,}192} = \frac{4{,}280}{8{,}192} = \frac{535}{1{,}024}$$

Since $Var(S^2) = \frac{115}{256} = \frac{460}{1{,}024}$, $Var(S^2) < Var(S_*^2)$.

(h) All the possible samples, with their corresponding midranges are:

$\{X_1, X_2, X_3\} =$

$= \{1,1,1\}$	with sample midrange	$M = \frac{1+1}{2} = 1$
$= \{1,1,0\}$	" " "	$M = \frac{1+0}{2} = 0.5$
$= \{1,0,1\}$	" " "	$M = \frac{1+0}{2} = 0.5$
$= \{0,1,1\}$	" " "	$M = \frac{1+0}{2} = 0.5$
$= \{1,0,0\}$	" " "	$M = \frac{1+0}{2} = 0.5$
$= \{0,1,0\}$	" " "	$M = \frac{1+0}{2} = 0.5$
$= \{0,0,1\}$	" " "	$M = \frac{1+0}{2} = 0.5$
$= \{0,0,0\}$	" " "	$M = \frac{0+0}{2} = 0$

Then:

$$P(M = 1) = P[\{1,1,1,\}] = f(1) \cdot f(1) \cdot f(1) = \frac{27}{64}$$

$$P(M = 0.5) = P[\{1,1,0\} \text{ or } \{1,0,1\} \text{ or } \{0,1,1\} \text{ or } \{1,0,0\} \text{ or } \{0,1,0\} \text{ or } \{0,0,1\}] =$$

$$= P[\{1,1,0\}] + P[\{1,0,1\}] + P[\{0,1,1\}] + P[\{1,0,0\}] + P[\{0,1,0\}] + P[\{0,0,1\}] =$$

$$= \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} =$$

$$= \frac{9}{64} + \frac{9}{64} + \frac{9}{64} + \frac{3}{64} + \frac{3}{64} + \frac{3}{64} = \frac{36}{64} = \frac{9}{16}$$

$$P(M=0) = P[\{0,0,0\}] = f(0)\cdot f(0)\cdot f(0) = \frac{1}{64}$$

So the pmf of the sample midrange from sample size n=3 is:

$$f(M=1) = \frac{27}{64}$$
$$f(M=0.5) = \frac{36}{64} = \frac{9}{16}$$
$$f(M=0) = \frac{1}{64}$$

$f(M) = 0$ otherwise

Then:

$$E(M) = \sum_i M_i f(M_i) = 1\cdot\frac{27}{64} + 0.5\cdot\frac{36}{64} + 0\cdot\frac{1}{64} = \frac{27}{64} + \frac{18}{64} = \frac{45}{64}$$

We know from the previous exercise that

$$E(X) = \frac{3}{4}$$

Since $E(M) \neq E(X)$, M is a biased estimator of $E(X)$.

The bias here is:

$$B = E(M) - E(X) = \frac{45}{64} - \frac{3}{4} = \frac{45}{64} - \frac{48}{64} = -\frac{3}{64}$$

(i) The variance of the sample midrange is:

$$V(M) = \sum_i [M_i - E(M)]^2 f(M_i) = (1-\frac{45}{64})^2\cdot\frac{27}{64} + (0.5-\frac{45}{64})^2\cdot\frac{36}{64} + (0-\frac{45}{64})^2\cdot\frac{1}{64} =$$
$$= (\frac{19}{64})^2\cdot\frac{27}{64} + (-\frac{13}{64})^2\cdot\frac{36}{64} + (-\frac{45}{64})^2\cdot\frac{1}{64} = \frac{9,747}{262,144} + \frac{6,084}{262,144} + \frac{2,025}{262,144} = \frac{17,856}{262,144} =$$
$$= \frac{279}{4,096} = 0.0681$$

Then the MSE of the sample midrange is:

$$MSE(M) = V(M) + Bias^2(M) = \frac{279}{4{,}096} + (-\frac{3}{64})^2 = \frac{279}{4{,}096} + \frac{9}{4{,}096} = \frac{288}{4{,}096} =$$

$$= \frac{9}{128} = 0.0703$$

From the sample mean theorem and from previous exercise we know that:

$$V(\overline{X}) = \frac{V(X)}{n} = \frac{3/16}{3} = \frac{3}{48}$$

and that the bias of $\overline{X}$ as an estimator of the population mean is zero. So:

$$MSE(\overline{X}) = V(\overline{X}) + Bias^2(\overline{X}) = \frac{3}{48} + 0 = \frac{3}{48} = 0.0625$$

We therefore have $MSE(M) > MSE(\overline{X})$

(Also note that $V(M) > V(\overline{X})$)

9. (i) We need to evaluate $P(Y > 10.223)$. To do so, we need to find the distribution of Y. Note that since Y is a linear function of two normal random variables, it also must be normal. To find its distribution we only need to find its mean and variance.

$$E(Y) = E\left(\frac{\log Y_3 + \log Y_2}{2}\right) = \frac{1}{2}E(\log Y_3) + \frac{1}{2}E(\log Y_2) =$$

$$= \frac{1}{2}\cdot 9.711 + \frac{1}{2}\cdot 9.702 = 9.7065$$

To find the variance of Y, we must first find the covariance between $\log Y_3$ and $\log Y_2$. Using the definition of correlation:

$$Cor(\log Y_3, \log Y_2) = \frac{Cov(\log Y_3, \log Y_2)}{\sqrt{V(\log Y_3)}\cdot\sqrt{V(\log Y_2)}} \Rightarrow$$

$$\Rightarrow Cov(\log Y_3, \log Y_2) = 0.73 \cdot 8.700 \cdot 8.666 = 55.038$$

Then:

$$V(Y) = V\left[\frac{1}{2}(\log Y_3 + \log Y_2)\right] = \frac{1}{4}V(\log Y_3) + \frac{1}{4}V(\log Y_2) + 2\cdot\frac{1}{2}\cdot\frac{1}{2}Cov(\log Y_3, \log Y_2) =$$

$$= \frac{1}{4}\cdot 75.682 + \frac{1}{4}\cdot 75.093 + \frac{1}{2}55.038 = 37.694 + 27.519 = 65.213$$

That is, $Y \sim N(9.7065, 65.213)$

Then:

$$P(Y > 10.223) = 1 - P(Y \leq 10.223) = 1 - P\left[\frac{Y - E(Y)}{\sqrt{V(Y)}} \leq \frac{10.223 - 9.7065}{\sqrt{65.213}}\right] =$$
$$= 1 - P(Z \leq 0.064) = 1 - \Phi(0.064) = 1 - 0.525 = 0.475$$

(ii) We have:

$$E(\overline{Y}) = E(Y) = 9.7065$$

$$V(\overline{Y}) = \frac{V(Y)}{n} = \frac{65.213}{n} = 21.738$$

and $\overline{Y}$ is normally distributed because the population we are sampling from is normal. Therefore:

$$\overline{Y} \sim N(9.7065, 21.738)$$

Then:

$$P(\overline{Y} > 10.223) = 1 - P(\overline{Y} \leq 10.223) = 1 - P\left[\frac{\overline{Y} - E(\overline{Y})}{\sqrt{V(\overline{Y})}} \leq \frac{10.223 - 9.7065}{\sqrt{21.738}}\right] =$$
$$= 1 - P(Z \leq 0.111) = 1 - \Phi(0.111) = 1 - 0.5438 = 0.4562$$