

Problems for Class 9

TRUE or FALSE problems

State whether you believe the given statement is TRUE or FALSE and provide a brief argument for your answer.

1. A confidence interval always includes the estimated population parameter.
2. Consider a 99% and a 90% confidence interval for the same population parameter estimated by the same estimator. Then the length of the 99% interval will always be greater than that of the 90% interval.
3. We have a random sample from a population with pmf depending on the parameter θ . Let T_1 and T_2 be two sample statistics from this random sample. We have:

$$P[T_1 \leq \theta \leq T_2] = \theta^2, \text{ for } \theta \leq 1.$$

Then interval $[T_1, T_2]$ is called a $100\theta^2$ per cent confidence interval for θ .

4. When sampling from a normal population with unknown mean and variance, the expected value of the standardised sample mean is always zero.
5. When sampling from a normal distribution, the expected value of the sample mean follows the t distribution.

Exercises

Exercises 1-5: NCT 8.10, 8.11, 8.16, 8.18, 8.25.

6. We have random samples of size 16, each from a different normal population with unknown means μ_i , $i=1, 2, 3$, and known variances equal to 4, 6, and 8 respectively.
(a) Construct a 90 percent confidence interval for each of the three means. What happens to the length of the interval as the population variance increases? What is the intuition behind this result?
(b) For the population with the variance equal to 4, construct two 90 per cent confidence intervals of the same length, and different from the one you found in (a).
7. [04-05 final] Suppose we are interested in the earnings of married couples in the UK and we know the following for 2003:

- a. the natural logarithm of men's earnings in these couples followed a normal distribution with mean 9.693 and variance 57.774;
- b. the natural logarithm of women's earnings in these couples followed a normal distribution with mean 9.680 and variance 53.483;
- c. the correlation of log earnings between men and women was 0.40.

(i) If you randomly draw one couple from this population of married couples, what is the probability that this couple's earnings (that is, the sum of the two spouses' earnings) were between £21,000 and £25,000?

(ii) If you draw a random sample of four couples out of this population, what is the probability that the average log earnings of these couples were between $\log(21,000)$ and $\log(25,000)$?

(iii) Construct a 90 percent confidence interval for the sample mean of the random sample in (ii).

8. A research firm conducted a survey to determine the mean amount students spent on drinks during a week. Suppose that it is known from other surveys that the distribution of these expenditures is normal and the variance is 25. A sample of 49 students was drawn.

(i) Construct a 95 per cent confidence interval for the population mean.

(ii). What would the sample size have to be for the length of the confidence interval to be 1?