

INTERPRETING COEFFICIENTS IN OLS OUTPUT – (1)

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This is a brief guide to interpreting coefficients for which the variables are non-linear. Remember, the parameters (the b) are always linear in OLS models, but the variables may not be. Indeed, we can transform variables in order to ensure they better fit OLS assumptions. Here the following models are discussed:

- (1) Linear $\rightarrow Y = a + bX$
- (2) Lin-Log $\rightarrow Y = a + b\ln X$
- (3) Log-Lin $\rightarrow \ln Y = a + bX$
- (4) Log-Log $\rightarrow \ln Y = a + b\ln X$
- (5) Quadratic Terms $\rightarrow Y = a + b(1)X + b(2)X^2$
- (6) For Dummy Variable Interpretation, see file “Interpreting Regression Output” on course web page on my website.

It looks like there are lots of equations, but don't worry. I have tried to show each step, so that it is simple and easy to follow.

(1) Linear $\rightarrow Y = a + bX$

- The basic OLS model used whenever the relationship between the regressand, Y, and the regressors can be assumed, or shown to be linear.
- $\frac{dY}{dX} = b$
- Increase X by 1 unit and Y will increase by b units.
- Remembering that an elasticity is the percentage or relative change in Y divided by the relative change in X, that is:

$$\varepsilon = \frac{\text{relative } \Delta Y}{\text{relative } \Delta X} = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y} = \frac{dY}{dX} \cdot \frac{X}{Y} = b \cdot \frac{X}{Y}, \text{ we can evaluate the}$$

elasticity at the mean values of X and Y so that:

$\varepsilon = b \cdot \frac{\bar{X}}{\bar{Y}}$ where a bar indicates the mean value. Remember, by construction a regression line passes through the X-bar and Y-bar.

- A 1% increase in X (at X-bar) increases Y by $\varepsilon\%$.

(2) Lin-Log $\rightarrow Y = a + b \ln X$

- A 1 unit increase in $\ln X$ will make Y increase by b.
- $\frac{dY}{d \ln X} = \frac{\text{absolute } \Delta Y}{\text{relative } \Delta X} = \frac{\Delta Y}{\Delta X / X} = \frac{\partial Y}{\partial X} \cdot X = b$

(Remember the derivative of $\ln X$ is $(1/X)$)

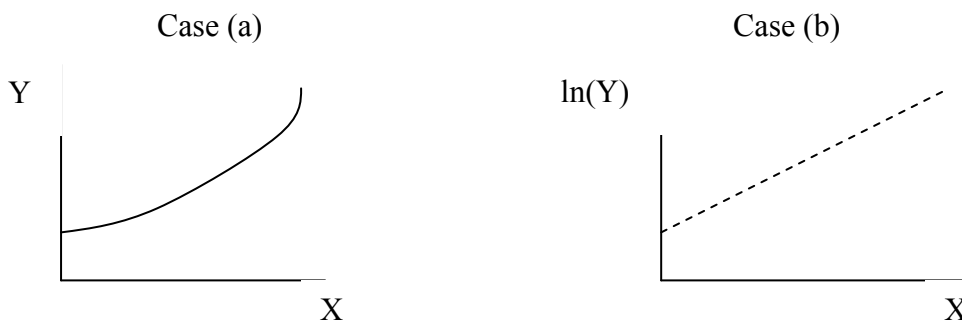
- To get an elasticity, we need to divide this by Y evaluated at the mean, y-bar.

$$b = \frac{\partial Y}{\partial X} \cdot \frac{X}{1} \implies \varepsilon = \frac{\partial Y}{\partial X} \cdot \frac{X}{1} \cdot \frac{1}{Y} = \frac{\partial Y}{\partial X} \cdot \frac{X}{Y} = \frac{b}{Y}$$

A 1% increase in X causes Y (at Y-bar) to increase by $\varepsilon\%$.

(3) Log-Lin $\rightarrow \ln Y = a + bX$

- Used when the relationship between the regressand and regressors, X, is exponential. Logging X ensures linearity (which we test using a J-test), so that we change from case (a) to case (b) below:



- $$\frac{d \ln Y}{dX} = \frac{\text{relative } \Delta Y}{\text{absolute } \Delta X} = \frac{\Delta Y / Y}{X} = \frac{\partial Y}{\partial X} \cdot \frac{1}{Y} = b$$

- Unsurprisingly, to get an elasticity, we have to multiply this by X (evaluated at the mean, X-bar) so that:

$$\varepsilon = b \times \bar{X}$$

- A 1% increase in X, evaluated at X-bar, causes Y to increase by $\varepsilon\%$.

(4) Log-Log $\rightarrow \ln Y = a + b \ln X$

- Often used to estimate consumption functions. The intercept, a, is autonomous consumption, and b is the (constant) marginal propensity to consume out of income.
- Note that b is already an elasticity.

(5) Quadratic Terms $\rightarrow Y = a + b(1)X + b(2)X^2$

where 2 indicates a square term.

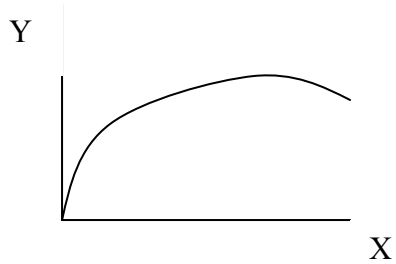
- This is done, typically, in models in which we believe there may be diminishing marginal returns. For example, age and age-square are often included in regressions estimating (log of) income (where age is a proxy for experience).

- $$\frac{d \ln Y}{dX} = b(1) + 2 \cdot b(2)X$$

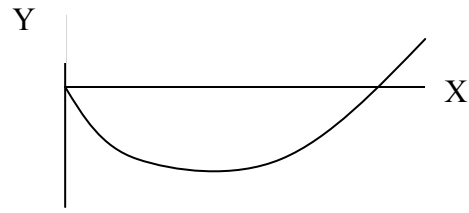
- This creates the following relationships in Y (assuming 0 constants):

Case (a)

Case (b)



For $b(1) > 0$
and $b(2) < 0$



For $b(1) < 0$
and $b(2) > 0$

Here, turning points are shown, but these may be outside of realistic values of X , making diminishing marginal returns (in case (a)) the most interesting part of the story.

- If calculating a turning point is interesting, for example, the age at which salary begins to decline, then set the derivative to zero and solve for X :

$$\frac{d \ln Y}{dX} = 0 = b(1) + 2 \bullet b(2)X \implies X = \frac{b(1)}{2 \bullet b(2)}$$